$\therefore x^2(1+3)=16$; ......$x=\pm_1 2$; $y=\pm_1 3$; $z=\pm_1 4$.

Test r_2......t_1, etc., by $(B)(C)$ gives $17(t^2+r)=22(r^2+t)$......(J).

Thus $r_1=0.125$......in (J) *fixes* $t_1=1.2073$ (4 true decimals).

$r_2=1.0217$ in (J) *fixes* $t_2=0.217739$ (6 true decimals).

$r_a=2.785434$ in (J) *fixes* $t_a=3.417229$ (6 true decimals).

$\therefore x_1=\pm_1$......to	$y_1=\pm_1$......to	$z_1=\pm_1$ .....
3.728406 657543 8	0.466309 516672 3	4.501267 432220 0
$x_2=\pm_1$......to	$y_2=\pm_1$......to	$z_2=\pm_1$......
4.536281 490681 969	4.634718 990123 0830	0.987729 704967 0174
$x_a=\pm_1$......to	$y_a=\pm_1$......to	$z_a=\pm_1$......
1.233342 644210 0772	3.435894 472231.4779	4.214615 222418.7456

Read $\pm_1$......$\pm_1$......$\pm_1$ "change in unison," not $\pm$ *prime*, $\mp$ *prime*.
One page finds *all* 24 roots of x, y, z. Q. V. D.

[*Prof. Charles C. Cross* remarks that two solutions are given in the *Mathematical Diary* for 1831, pages 150-151, and that there is quite an interesting history of this "curious and important question" given by "Unicorn," of North Carolina.

87. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

A starts to travel around a circular island at a given point and travels at the rate of 5 miles in 4 hours. One half hour after A, B starts from a point directly opposite from A and travels in an opposite direction at the rate of 4 miles in 3 hours. One hour afterwards C starts from the same point as A and travels in an opposite direction to A at the rate of 3 miles in 2 hours. One half hour afterwards D starts from the same point as B and travels in an opposite direction to B at the rate of 2 miles in 1 hour. Required the size of the island, and when they will all be together, and how far each will have traveled at the accomplishment of this event.

Solution by the PROPOSER.

This problem furnishes a good example in indeterminate analysis. Of course an unlimited number of answers may be obtained each of which will satisfy the conditions of the problem ; but we shall be content to show in what manner one set may be obtained.

A's rate of travel equals $1\frac{1}{4}$ miles per hour, B's rate equals $1\frac{1}{3}$ miles per hour, C's rate equals $1\frac{1}{2}$ miles per hour, and D's rate equals 2 miles per hour. B starts $\frac{1}{2}$ hour after A starts, C starts $1\frac{1}{2}$ hours after A, and D starts 2 hours after A.

Let c=the circumference of the island; then A and B will be together for first time after A starts in $[(\frac{1}{2}c-1\frac{1}{4}\times\frac{1}{2})/(1\frac{1}{4}+1\frac{1}{3})]+\frac{1}{2}=(6c+8)/31$ hours.

For the second time in $[(1\frac{1}{2}c-1\frac{1}{4}\times\frac{1}{2})/(1\frac{1}{4}+1\frac{1}{3})]+\frac{1}{2}=(18c+8)/31$ hours ; and for the $(m+1)$th time in $[(6+12m)c+8]/31$ hours......(A).

A and C will be together for the first time after A starts in $[(c-1\frac{1}{2}\times1\frac{1}{4})/(1\frac{1}{2}+1\frac{1}{4})]+1\frac{1}{2}=(4c+9)/11$ hours.

For the second time in $[(2c-1\frac{1}{2}\times1\frac{1}{4})/(1\frac{1}{2}+1\frac{1}{4})]+1\frac{1}{2}=(8c+9)/11$ hours ; and for the $(n+1)$th time in $[(4+4n)c+9]/11$ hours......(B).

A and D will be together for the first time after A starts in $[(\frac{1}{2}c+1\frac{1}{4}\times 2)/(2-1\frac{1}{4})]+2=(2c+16)/3$ hours.

For the second time in $[(1\frac{1}{2}c+1\frac{1}{4}\times 2)/(2-1\frac{1}{4})]+2=(6c+16)/3$ hours, and for the $p+1$ time in $[(2+4p)c+16]/3$ hours .....(C).

Now when the four persons are all together, the general expressions (A), (B), and (C) will represent the same number of hours. Hence, at the occurrence of such an event, we may equate them, and we shall have

$[(6+12m)c+8]/32=[(4+4n)c+9]/11$......(1), and

$[(4+4n)c+9]/11=[(2+4p)c+16]/3$......(2).

From equation (1) we obtain $c=191/(132m-124n-68)$......(3).

From equation (2) we obtain $c=149/(12n-44p-10)$......(4).

Equating (3) and (4), clearing of fractions, and dividing the resulting equations by 44, we obtain $447m-472n+191p=153$ .....(5).

Here we have three unknown quantities to be determined from a single equation, but as their values are necessarily restricted to whole numbers, we determine one set of such values as follows: Give to one of the unknown quantities any integral value we choose; as 8 is a convenient value, put $p=8$, and (5) will become $447m-472n=-1375$......(6), or $m=n+(25n-1375)/447$.....(7). Now as the fractional expression $(25n-1375]/447$ must be a whole number, in order to make m such, give to n any integral value in it that shall render the expression a positive quantity. For instance, put $n=70$ and the expression will become $\frac{375}{447}$. We perceive that n must be increased to give an integral result, and by how much we determine as follows:

$(25\times 447-375)/25=332$. $\therefore$ if we take $n=432+70=502$ we shall find that the expression will equal 25, a whole number, and by substitution in (7) we shall find $m=502+25=527$. $\therefore$ the values $p=8$, $m=527$, and $n=502$ will satisfy equation (5). Substituting these values in equations (3) or (4), we have $c=\frac{1}{38}$ of a mile, the circumference of the island. Again, substituting these values found for m, n, p, and c in either of the expressions (A), (B), or (C), we find that A, B, C, and D will be altogether in $5\frac{12}{19}$ hours after A starts.

Proof and remaining answers. In $5\frac{12}{19}$ hours A will travel $\frac{107}{19}\times\frac{5}{4}=7\frac{3}{76}$ miles, and he will go around the island $7\frac{3}{76}\div\frac{1}{38}=262\frac{1}{2}$ times, and hence will be at B's starting point at the end of the time. B, who travels $\frac{1}{2}$ hour less than A, or $5\frac{12}{19}-\frac{1}{2}=\frac{195}{38}$ hours, will travel $\frac{195}{38}\times\frac{4}{3}=6\frac{6}{19}$ miles, and he will go around the island $6\frac{6}{19}\div\frac{1}{38}=260$ times, and hence will be at his starting point at the end of the time. C, who travels $1\frac{1}{2}$ hours less than A, or $5\frac{12}{19}-1\frac{1}{2}=\frac{157}{38}$ hours, will travel $\frac{157}{38}\times\frac{3}{2}=6\frac{15}{76}$ miles, and he will go around the island $6\frac{15}{76}\div\frac{1}{38}=235\frac{1}{2}$ times, and hence will be at B's starting point at the end of the time. D, who travels 2 hours less than A, or $5\frac{12}{19}-2=3\frac{12}{19}$ hours, will travel $3\frac{12}{19}\times 2=7\frac{5}{19}$ miles, and he will go around the island $7\frac{5}{19}\div\frac{1}{38}=276$ times, and he will be at B's starting point at the end of the time. As they will all be at B's starting point at the end of $5\frac{12}{19}$ hours after A starts, they must all be together in that time.